Chapter

2

Fraction Operations

GOAL

You will be able to

- multiply and divide fractions by whole numbers, other fractions, and mixed numbers using models, drawings, and symbols
- estimate products and quotients of whole numbers, fractions, and mixed numbers
- solve and create problems using fraction operations
- calculate the value of expressions involving fractions, using the proper order of operations
- communicate clearly about fraction operations

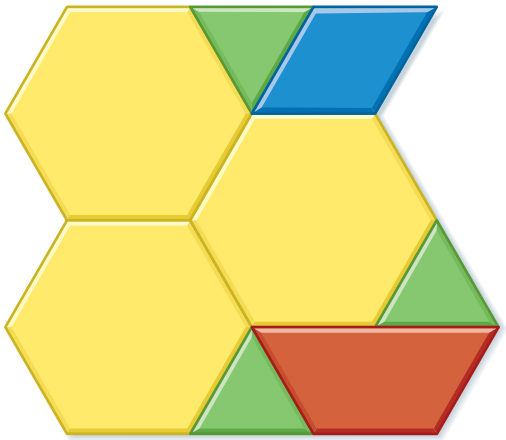
◀ How could you use fractions to describe this "musical instrument"?

Getting Started

YOU WILL NEED
- pattern blocks

Pattern Block Designs

Allison made a design using pattern blocks.



What fractions can you use to describe the pattern block design?

A. If a yellow hexagon has an area of 1 unit, what is the area of each block?

a) the red block **b)** the blue block **c)** a green block

B. The equation $3 + \frac{3}{6} = 3\frac{1}{2}$ tells the sum of the areas of two of the colours.
- Which are the two colours?
- How do you know?

C. The equation $3 - \frac{1}{2} = 2\frac{1}{2}$ describes how much more of the design is one colour than another.
- Which are the two colours?
- How do you know?

D. Write equations with fractions and/or **mixed numbers** to describe the areas defined below, using the units in part A. Solve the equations. Show your work.
- the red and blue parts
- how much more is green than blue
- how much more is yellow and red than green and blue
- how much more is red than green

E. Write three other fraction equations that describe areas in Allison's design.

F. Make your own design using
- yellow, red, blue, and green pattern blocks
- a total of eight blocks
- at least two yellow blocks
- at least one block of each other colour

Repeat steps D and E for your design.

G. Is it possible to create a design using the rules in step F where each is true? Explain.
- The yellow area is $1\frac{1}{3}$ units greater than the blue area.
- The blue and red area, together, is $\frac{1}{6}$ unit greater than the green area.

What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.

1. If you add two fractions, the result is always less than 1, but if you add three fractions, it is greater than 1.

2. If you subtract two fractions, the difference is usually somewhere between the two fractions you are subtracting.

3. The product of two numbers is always greater than the sum.

4. The quotient of two numbers is always less than the product.

5. One way to calculate $a \div b$ is to figure out how many bs make up a.

2.1 Multiplying a Whole Number by a Fraction

YOU WILL NEED

- grid paper
- counters
- Fraction Strips Tower
- Number Lines

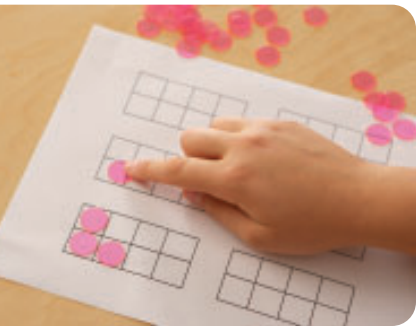
GOAL

Use repeated addition to multiply fractions by whole numbers.

LEARN ABOUT the Math

Nikita is having a party. After a few hours, she notices that six pitchers of lemonade are each only $\frac{3}{8}$ full. She decides to combine the leftovers to use fewer pitchers.

How many pitchers will the leftover lemonade fill?



A. Estimate how many whole pitchers the lemonade will fill completely. Explain your thinking.

B. Use a model to represent $\frac{8}{8}$ of a pitcher.

C. Use this model to represent all the lemonade from the six partially full pitchers. Write the number of pitchers, after the lemonade has been combined, as both an **improper fraction** and a **mixed number**.

D. Why could you write either $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$ or $6 \times \frac{3}{8}$ to describe the total amount of lemonade in the pitchers?

E. Now use another model to represent the pitchers and solve the problem.

Reflecting

F. How could you have predicted that the amount left in the last pitcher would be a fraction with a denominator of 8?

G. Describe a procedure for multiplying a whole number by a fraction. Explain why you think that procedure will work.

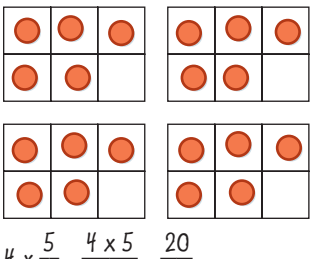
WORK WITH the Math

Example 1 | Multiplying with grids and counters

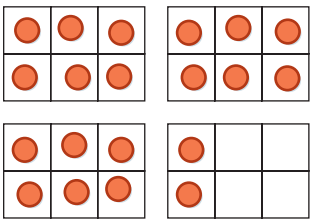
Calculate $4 \times \frac{5}{6}$ using grids and counters.
Represent the answer as an improper fraction and as a mixed number.

Brian's Solution

$4 \times \frac{5}{6}$ is four sets of $\frac{5}{6}$.



$4 \times \frac{5}{6} = \frac{4 \times 5}{6} = \frac{20}{6}$



$4 \times \frac{5}{6} = \frac{20}{6}$, or $3\frac{2}{6}$, or $3\frac{1}{3}$

I used 3-by-2 rectangles, since I wanted to show sixths and $3 \times 2 = 6$. Each rectangle represents one whole.

I showed four sets of $\frac{5}{6}$ by putting counters on 5 out of 6 squares in each of the four rectangles.

$4 \times 5 = 20$, so 20 squares are covered.

Since each square represents $\frac{1}{6}$, the 20 squares represent $\frac{20}{6}$.

To write the improper fraction as a mixed number, I moved the counters to fill as many grids as I could.

I moved 3 counters from the last grid to fill up the other 3 grids.

That means 3 grids were full and there were 2 counters, each representing $\frac{1}{6}$, in the last grid.

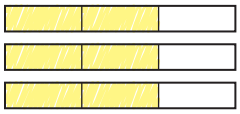
You can write the fraction part $\frac{2}{6}$ as $\frac{1}{3}$ if you want to.

Example 2 | Multiplying with fraction strips

Calculate $3 \times \frac{2}{3}$ using fraction strips. Write the product as an improper fraction and as a whole or mixed number.

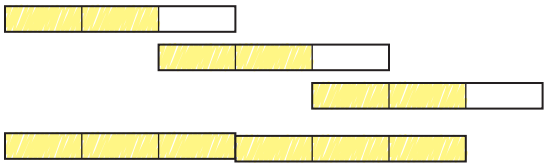
Misa's Solution

$3 \times \frac{2}{3}$ is 3 sets of $\frac{2}{3}$.



$3 \times \frac{2}{3} = \frac{3 \times 2}{3} = \frac{6}{3}$

I can look at the model and see that there are 6 thirds altogether.



$\frac{3 \times 2}{3} = 2$

To rename the product as a whole or mixed number, I needed to know how many whole strips there were and how many thirds were left over. I lined up the strips to see.

The total length matched 2 full strips and there were no extra thirds

Example 3 | Multiplying with a number line

Calculate $5 \times \frac{3}{2}$ using a number line. Write the product as an improper fraction and as a whole or mixed number.

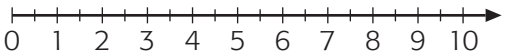
Preston's Solution

$5 \times \frac{3}{2} = 5 \times 3$ halves

$= 15$ halves

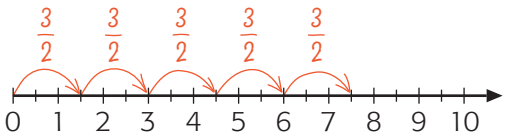
$= \frac{15}{2}$

I knew that there would be 15 halves since there are 5 sets of 3 halves.



I drew a number line marked in halves to see how much $\frac{15}{2}$ is.

I knew it would be less than 10 since $\frac{20}{2} = 10$.



$5 \times \frac{3}{2} = 5 \times 1\frac{1}{2}$

$= 7\frac{1}{2}$

I thought of $\frac{3}{2}$ as $1\frac{1}{2}$. I made 5 jumps of $1\frac{1}{2}$ and ended up at $7\frac{1}{2}$.

That makes sense since $\frac{15}{2}$ is 7 sets of 2 halves and another $\frac{1}{2}$. Each set of 2 halves is one whole.

Reading Strategy

Questioning

Write three questions that can help you solve this problem.

A Checking

1. Jennifer pours $\frac{2}{3}$ of a cup of water into a pot and repeats this 7 times. How many cups of water, in total, does she pour into the pot? Write your answer as a mixed number.

2. **a)** Write $5 \times \frac{3}{4}$ as a repeated addition sentence.
b) Use a model to calculate the answer.
c) Write your answer as an improper fraction and as a mixed number.

B Practising

3. Multiply. Write your answer as a fraction and, if it is greater than 1, as a mixed number or whole number. Use a model and show your work for at least two parts.

a) $2 \times \frac{1}{3}$ **c)** $6 \times \frac{3}{8}$ **e)** $3 \times \frac{7}{6}$

b) $5 \times \frac{3}{5}$ **d)** $4 \times \frac{2}{5}$ **f)** $8 \times \frac{4}{2}$

4. Estimate to decide which products are between 5 and 10. Calculate to check.

a) $2 \times \frac{5}{3}$ **c)** $8 \times \frac{4}{5}$ **e)** $6 \times \frac{5}{4}$

b) $6 \times \frac{6}{7}$ **d)** $7 \times \frac{7}{10}$ **f)** $7 \times \frac{7}{6}$

5. Art class is $\frac{5}{6}$ of an hour each school day. How many hours of art does a student have in five days?

6. Jason needs $\frac{2}{3}$ of a cup of flour to make one batch of bannock. How many cups of flour will he need if he decides to make six batches, one for each of his aunts?

7. Katya says that multiplying $17 \times \frac{1}{4}$ will tell her how many dollars 17 quarters is worth. Do you agree? Explain.

8. **a)** How much farther are four jumps of $\frac{3}{5}$ on a number line than three jumps of $\frac{4}{5}$? Explain.

b) Select two other pairs of jumps that would be the same distance apart as the jumps in part a).

9. **a)** Multiply $2 \times \frac{2}{5}$.

b) Rewrite $\frac{2}{5}$ as a percent, and then multiply by 2.

c) Explain how you can use the calculation in part b) to check your answer to part a).

10. Multiply 5×2.3. Show how to use fraction multiplication to check your result.

11. Ki multiplied a whole number by a fraction. The numerator of the fraction product was 30. List three possible whole number and fraction combinations he could have been using.

12. Carmen multiplied a whole number by a fraction. Her answer was between 6 and 8. List three possible multiplications Carmen might have performed.

13. Describe a situation where you might multiply $4 \times \frac{2}{3}$.

14. Lea modelled the product of $5 \times \frac{\square}{\square}$ using grids and counters and filled exactly four grids. What fraction did she multiply by?

15. At a party, Raj notices that 15 pitchers of lemonade are filled to the same level, but not to the top. He combines all the lemonade to fill six whole pitchers. What fraction of each of the 15 pitchers was full?

16. **a)** Why do the products for $5 \times \frac{2}{3}$, $5 \times \frac{2}{5}$, and $5 \times \frac{2}{7}$ all have the same numerator?

b) Why are the denominators different?

2.2 Exploring Calculating a Fraction of a Fraction

YOU WILL NEED

- Fraction Strips Tower
- scissors
- pencil crayons

GOAL

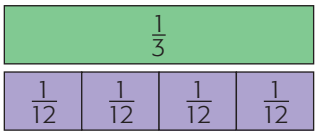
Represent one fraction as part of another fraction.

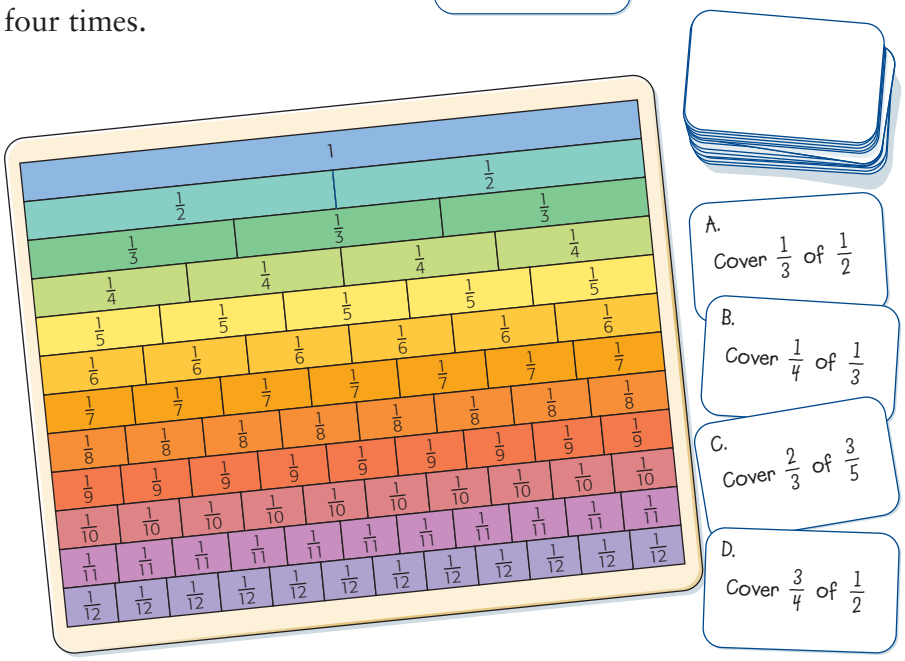
EXPLORE the Math

Aaron is playing a fraction game with his friends.
The game board is a fraction strip tower.
Each player picks a card and covers sections of fraction strips.
For example, if the player picks

B. Cover $\frac{1}{4}$ of $\frac{1}{3}$

, the player would cover $\frac{1}{12}$, since $\frac{1}{12}$ fits into $\frac{1}{3}$ four times.





? Which cards might Aaron pick from the deck to cover each of $\frac{1}{6}$, $\frac{2}{5}$, and $\frac{3}{8}$?

2.3 Multiplying Fractions

YOU WILL NEED

- Fraction Strips Tower
- grid paper
- coloured pencils

GOAL

Multiply two fractions less than 1.

LEARN ABOUT the Math

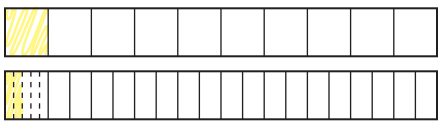
About $\frac{1}{10}$ of Canadians who are 12 and older downhill ski. About $\frac{2}{5}$ of these skiers are between the ages of 12 and 24.

? What fraction of the Canadian population between the ages of 12 and 24 are downhill skiers?

Example 1 | Using a fraction strip model

I needed to determine $\frac{2}{5}$ of $\frac{1}{10}$.

Allison's Solution



I used fraction strips.

I divided $\frac{1}{10}$ into 5 equal sections. I realized that, if I divided each $\frac{1}{10}$ up, that would make 50 equal sections. I coloured $\frac{2}{5}$ of the first tenth.

So 2 of 50 sections were coloured.

I could write that as $\frac{2}{50}$.

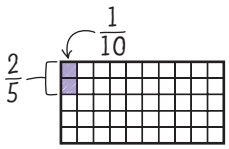
$\frac{2}{5}$ of $\frac{1}{10} = \frac{2}{50}$

About $\frac{2}{50}$ of Canadians between the ages of 12 and 24 downhill ski.

Example 2 | Using a grid

I calculated $\frac{2}{5} \times \frac{1}{10}$.

Nikita's Solution



Since ■ groups of ■ is another way of saying ■ × ■, I figured that $\frac{2}{5}$ of $\frac{1}{10}$ would be $\frac{2}{5} \times \frac{1}{10}$.

One way to multiply whole numbers is to draw a rectangle with those dimensions and calculate its area.

To show fifths, I wanted the rectangle to have 5 sections in one direction. To show tenths, I wanted it to have 10 sections in the other direction. So I made a 5-by-10 rectangle. Inside of it, I drew a rectangle that was $\frac{2}{5} \times \frac{1}{10}$. Its area was 2 × 1 squares out of the total 5 × 10 squares.

$$\frac{2}{5} \times \frac{1}{10} = \frac{2 \times 1}{5 \times 10}$$
$$= \frac{2}{50}$$
$$= \frac{1}{25}$$

I noticed that 2 and 50 had a common factor, so I wrote the product in lower terms.

Reflecting

A. How did Nikita's model show both $\frac{2}{5} \times \frac{1}{10}$ and $\frac{1}{10} \times \frac{2}{5}$?

B. How can you use a model to determine the numerator and denominator of a product?

C. Suggest a possible procedure for multiplying two fractions less than 1. Explain why you chose that procedure.

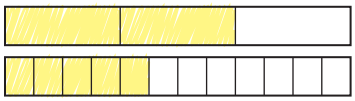
WORK WITH the Math

Example 3 | Multiplying fractions less than 1

If $\frac{2}{3}$ of the students in Windham Ridge School are in Grades 7 and 8, and if $\frac{5}{8}$ of these students are girls, what fraction of the students in the school are girls in Grades 7 and 8?

Solution A: Using fraction strips

This model shows $\frac{5}{8}$ of $\frac{2}{3}$.



$\frac{5}{8} \times \frac{2}{3} = \frac{5}{12}$

So $\frac{5}{12}$ of the students are girls in Grades 7 and 8.

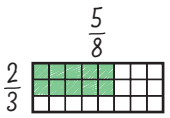
To take $\frac{5}{8}$ of $\frac{2}{3}$, you need a strip that divides the $\frac{2}{3}$ into 8 sections.

A strip to divide each third into 4 sections would work.

Use twelfths since $3 \times 4 = 12$.

Divide $\frac{2}{3}$ into 8 equivalent sections, and colour 5 of the sections.

Solution B: Using an area model



$\frac{5}{8} \times \frac{2}{3} = \frac{5 \times 2}{8 \times 3}$

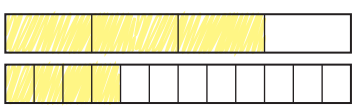
$= \frac{10}{24}$, or $\frac{5}{12}$

So $\frac{5}{12}$ of the students are girls in Grades 7 and 8.

Colour a 3-by-8 rectangle to show $\frac{5}{8}$ by $\frac{2}{3}$.

A Checking

1. What multiplication expression does each model represent?

a) 

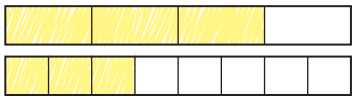
b) 

2. Draw a model for $\frac{3}{4} \times \frac{2}{5}$. Use your model to determine the product.

3. About $\frac{2}{11}$ of Canadian downhill skiers are from British Columbia. About $\frac{1}{10}$ of Canadians downhill ski. What fraction of all Canadians are downhill skiers from British Columbia?

B Practising

4. What multiplication expressions does each model represent?

a) 

b) 

c) 

5. Draw a model for each multiplication expression. Determine the product. Write the result in lowest terms.

a) $\frac{1}{2} \times \frac{3}{8}$

b) $\frac{4}{5} \times \frac{1}{3}$

c) $\frac{1}{6} \times \frac{2}{5}$

d) $\frac{3}{4} \times \frac{2}{6}$

e) $\frac{1}{4} \times \frac{4}{5}$

f) $\frac{3}{5} \times \frac{2}{3}$

6. Match each expression with its product in the box.

a) $\frac{5}{6} \times \frac{7}{10}$

b) $\frac{3}{8} \times \frac{4}{9}$

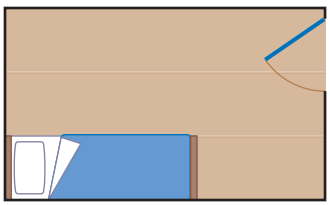
c) $\frac{2}{6} \times \frac{9}{10}$

d) $\frac{4}{7} \times \frac{14}{15}$

$\frac{1}{6}$	$\frac{8}{15}$
$\frac{3}{10}$	$\frac{7}{12}$

7. a) Draw a picture to show why $\frac{2}{5} \times \frac{3}{8} = \frac{6}{40}$.

b) List two other pairs of fractions with a product of $\frac{6}{40}$.



8. Matthew's bed takes up $\frac{1}{3}$ of the width of his bedroom and $\frac{3}{5}$ of the length. What fraction of the floor area does the bed use up?

9. Jessica is awake for $\frac{2}{3}$ of the day. She spends $\frac{5}{8}$ of that time at home.

a) What fraction of the day is Jessica awake at home?

b) How many hours is Jessica awake at home?

10. The Grade 8 class raised $\frac{2}{5}$ of the money to support the school's winter production. The Grade 8 boys raised $\frac{2}{3}$ of the Grade 8 money. What fraction of the whole production fund did the Grade 8 boys raise?

11. **a)** In Manitoba, Francophones make up about $\frac{1}{20}$ of the population. Only about $\frac{1}{8}$ of Francophones in Manitoba are under 15. What fraction of Manitoba's total population is made up of Francophones under 15?

b) Aboriginal peoples make up about $\frac{3}{20}$ of the population of Manitoba. Of those, only about $\frac{1}{3}$ are under 15. What fraction of Manitoba's total population is made up of Aboriginal peoples under 15?

12. Cheyenne gets home after 4 p.m. on school days about $\frac{1}{2}$ of the time. She gets home after 5 p.m. on about $\frac{2}{5}$ of those days. On what fraction of school days does she get home after 5 p.m.?

13. Describe a situation where you might multiply $\frac{3}{5} \times \frac{2}{3}$.

14. **a)** Complete this pattern and continue it for three more products.

$4 \times \frac{1}{2} =$ ■ $\quad 2 \times \frac{1}{2} =$ ■ $\quad 1 \times \frac{1}{2} =$ ■ $\quad \frac{1}{2} \times \frac{1}{2} =$ ■

b) How does this pattern explain the product of $\frac{1}{2} \times \frac{1}{2}$?

15. How much greater is the first product than the second?

a) $\frac{2}{7} \times \frac{2}{5}$ than $\frac{1}{7} \times \frac{3}{5}$

b) $\frac{3}{8} \times \frac{4}{9}$ than $\frac{1}{8} \times \frac{2}{3}$

c) $\frac{3}{5} \times \frac{2}{3}$ than $\frac{1}{5} \times \frac{1}{4}$

16. **a)** Calculate 0.4×0.3.

b) Rename each decimal as a fraction and multiply. What do you notice?

17. Use a pattern to help you determine the product of $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{99}{100}$.

18. How does the product of two fractions less than 1 compare to the two fractions? Is the product equal to, greater than, or less than each fraction? How do you know?

19. Daniel multiplied $\frac{3}{5}$ by another fraction less than 1.

a) What do you know about the denominator of the product?

b) What do you know about the numerator of the product?

2.4 Exploring Estimating Fraction Products

YOU WILL NEED

- fraction models
- Fraction Spinner

GOAL

Estimate to predict whether a fraction product is closer to 0, $\frac{1}{2}$, or 1.

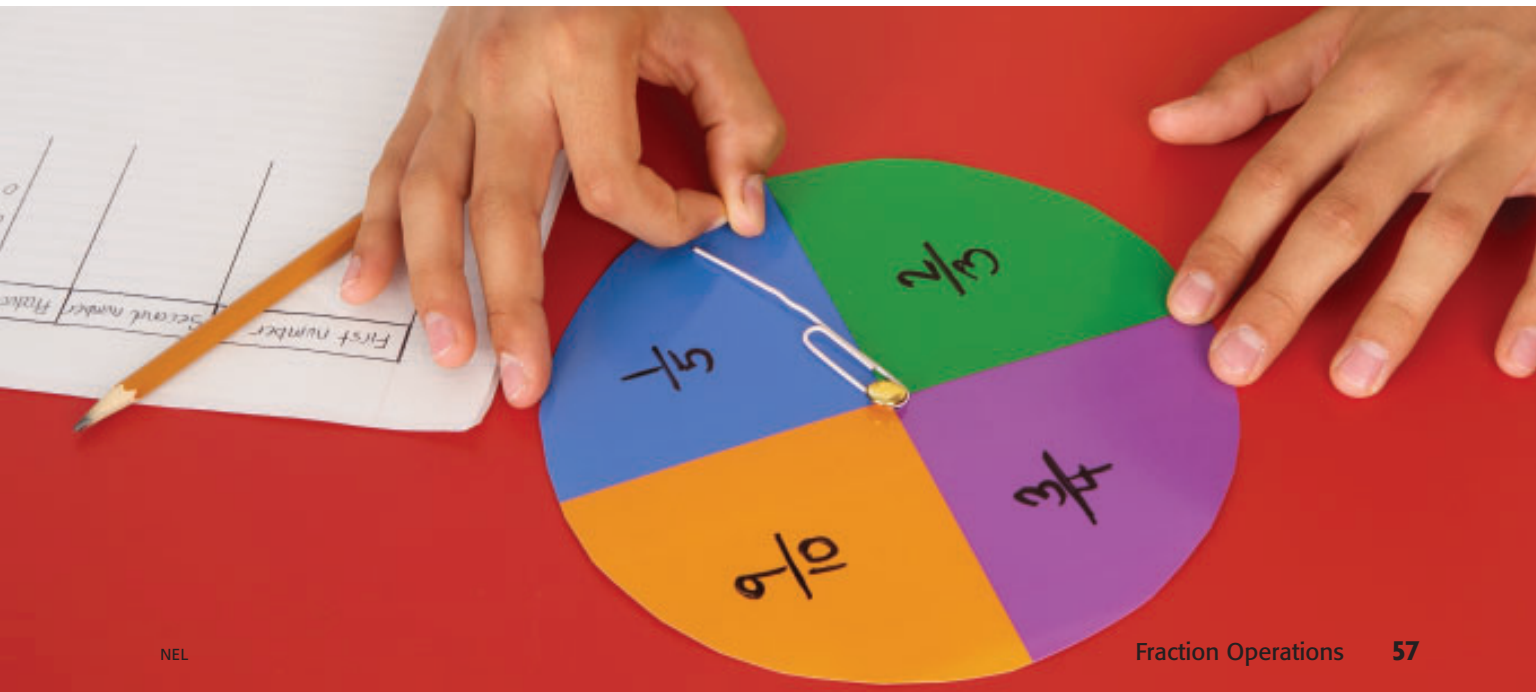
EXPLORE the Math

Brian and Preston are playing a spinner game. Brian is getting ready to spin.

Game Rules

Spin twice and multiply.
Score 1 point if the fraction is closer to $\frac{1}{2}$ than 0.
Score 1 more point if the fraction is closer to $\frac{1}{2}$ than 1.

? What combinations can Brian spin to win 2 points?



2.5 Multiplying Fractions Greater Than 1

YOU WILL NEED

- grid paper

GOAL

Multiply mixed numbers and improper fractions.

LEARN ABOUT the Math

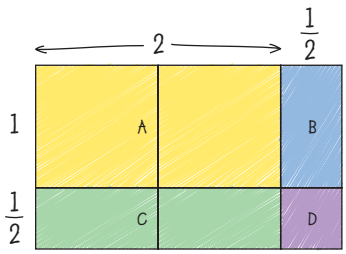
A large bag of popcorn holds $2\frac{1}{2}$ times as much as a small bag.
Aaron has $1\frac{1}{2}$ large bags.
He is pouring the popcorn into smaller bags to give to friends.

How many small bags will his popcorn fill?

Example 1 | Adding partial areas

I used an area model.

Aaron's Solution



The area of A is $1 \times 2 = 2$ square units.

The area of B is $1 \times \frac{1}{2} = \frac{1}{2}$ square unit.

The area of C is $\frac{1}{2} \times 2 = 1$ square unit.

The area of D is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ square unit.

The total area is $2 + \frac{1}{2} + 1 + \frac{1}{4}$ square units, or $3\frac{3}{4}$ square units.

He could fill $3\frac{3}{4}$ small bags of popcorn.

I knew this was a multiplication problem since I wanted $1\frac{1}{2}$ groups of $2\frac{1}{2}$.

One way to multiply is to get the area of a rectangle with side lengths the numbers you are multiplying. I drew a rectangle with side lengths $1\frac{1}{2}$ and $2\frac{1}{2}$.

I divided the rectangle into parts and calculated the area of each part.

I added up the partial areas.

Example 2 | Applying a procedure

I used a procedure.

Misa's Solution

$$1\frac{1}{2} \times 2\frac{1}{2} = \frac{3}{2} \times \frac{5}{2}$$
$$= \frac{3 \times 5}{2 \times 2}$$
$$= \frac{15}{4}$$

Area is $\frac{15}{4} = 3\frac{3}{4}$

He could fill $3\frac{3}{4}$ small bags of popcorn.

I knew this was a multiplication problem. When you multiply fractions less than 1, you can multiply the numerators and multiply the denominators.

I renamed $1\frac{1}{2}$ as $\frac{3}{2}$ and $2\frac{1}{2}$ as $\frac{5}{2}$ and multiplied the way I would multiply fractions less than 1.

Reflecting

A. Would you use Aaron's or Misa's method to multiply $1\frac{2}{3} \times 1\frac{4}{5}$? Explain your reasons.

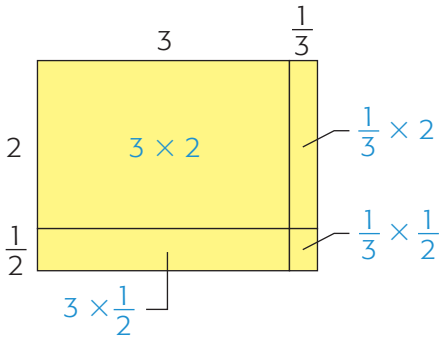
B. How would you use each model to multiply $2\frac{1}{3} \times 3\frac{1}{2}$?

WORK WITH the Math

Example 3 | Multiplying two mixed numbers

Multiply $2\frac{1}{2} \times 3\frac{1}{3}$.

Solution A: Adding partial areas

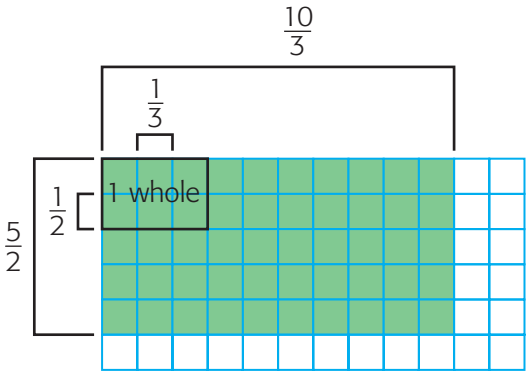


Calculate the area of the rectangle by calculating the four partial areas and then adding.

$$
\begin{aligned}
2\frac{1}{2} \times 3\frac{1}{3} &= \left(2 \times 3\frac{1}{3}\right) + \left(\frac{1}{2} \times 3\frac{1}{3}\right) \\
&= (2 \times 3) + \left(2 \times \frac{1}{3}\right) + \left(\frac{1}{2} \times 3\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) \\
&= 6 + \frac{2}{3} + \frac{3}{2} + \frac{1}{6} \\
&= 6 + \frac{4}{6} + 1 + \frac{3}{6} + \frac{1}{6} \\
&= 7 + \frac{8}{6} \\
&= 8\frac{2}{6} \text{ or } 8\frac{1}{3}
\end{aligned}
$$

Solution B: Using grids

$2\frac{1}{2} = \frac{5}{2}$ and $3\frac{1}{3} = \frac{10}{3}$



The dimensions are $\frac{5}{2} \times \frac{10}{3}$.

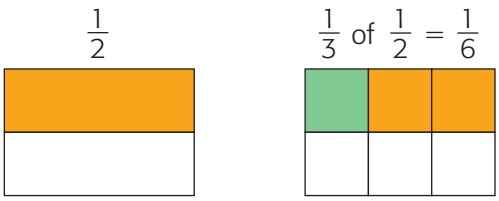
There are 5 x 10 = 50 squares.

Each square is $\frac{1}{6}$ of 1 whole.

The area is $\frac{50}{6}$.

$$2\frac{1}{2} \times 3\frac{1}{3} = \frac{5}{2} \times \frac{10}{3}$$
$$= \frac{5 \times 10}{2 \times 3}$$
$$= \frac{50}{6}, \text{ or } 8\frac{2}{6}, \text{ or } 8\frac{1}{3}$$

If you were multiplying $\frac{1}{2} \times \frac{1}{3}$, you would draw 2-by-3 grids.



Start with 2-by-3 grids to represent 1 whole.

Then extend the grid in both directions so there is space to draw a rectangle that is 5 halves wide and 10 thirds long.

A Checking

1. Estimate each product.

a) $\frac{5}{8} \times 6\frac{1}{2}$ **b)** $7\frac{2}{9} \times 6\frac{3}{4}$

2. Show each multiplication using a different model. Determine the product.

a) $2\frac{2}{3} \times 1\frac{4}{5}$ **b)** $\frac{1}{2} \times 4\frac{4}{7}$

3. Miriam is making $3\frac{1}{2}$ dozen cookies. If $\frac{2}{7}$ of the cookies have icing, how many dozen cookies have icing?

B Practising

4. Calculate each product.

a) $\frac{2}{3} \times 2\frac{1}{4}$ **c)** $\frac{1}{5} \times 2\frac{2}{3}$ **e)** $\frac{5}{6} \times 1\frac{5}{7}$

b) $\frac{5}{8} \times 1\frac{1}{2}$ **d)** $\frac{3}{4} \times 2\frac{5}{6}$ **f)** $\frac{2}{9} \times 1\frac{1}{6}$

5. Use a model to show $2\frac{3}{4} \times 4\frac{1}{3}$. Then calculate the product.

6. Draw a sketch to show a model for each multiplication.

a) $\frac{4}{3} \times \frac{3}{2}$ **b)** $\frac{1}{4} \times 4\frac{4}{7}$ **c)** $2\frac{1}{5} \times 3\frac{1}{6}$

7. Calculate each product. Write the fraction parts in lowest terms.

a) $2\frac{1}{4} \times 3\frac{1}{3}$ **c)** $1\frac{4}{5} \times 2\frac{2}{3}$ **e)** $3\frac{1}{6} \times 2\frac{2}{3}$

b) $1\frac{1}{2} \times 2\frac{1}{2}$ **d)** $3\frac{1}{5} \times 2\frac{1}{4}$ **f)** $1\frac{1}{6} \times 1\frac{1}{4}$

8. A muesli recipe requires $1\frac{1}{4}$ cups of oatmeal. How many cups of oatmeal do you need for each number of batches?

a) $2\frac{1}{2}$ batches **b)** $3\frac{1}{3}$ batches

9. Zoë had $3\frac{1}{3}$ times as much money as her brother. She spent $\frac{2}{5}$ of her money on a new CD player. Now how many times as much money as her brother does Zoë have?

10. Tai calculated $3\frac{1}{3} \times 4\frac{3}{8}$. He multiplied the whole number parts together and then the fraction parts together to get an incorrect product of $12\frac{3}{24}$.

a) Why would estimation not help Tai realize he made a mistake?

b) How could you show Tai that his answer is incorrect?

11. Andrea's bedroom is $1\frac{1}{3}$ times as long as Kit's bedroom and $1\frac{2}{3}$ times as wide. What fraction of the area of Kit's bedroom is the area of Andrea's bedroom?

Reading Strategy

Visualizing

In your mind, create a picture of the information in this problem.

12. The highest point in Alberta is Mount Columbia. Mount Columbia is about $4\frac{3}{5}$ times as high as the highest point in New Brunswick, Mount Carleton. Mount Carleton is about $5\frac{3}{4}$ times as high as the highest point in Prince Edward Island. Compare the height of Mount Columbia to the highest point in Prince Edward Island.

13. **a)** Multiply $3\frac{4}{10}$ by $2\frac{3}{10}$.

b) Rename these two fractions as decimals, and multiply the decimals.

c) How was the decimal multiplication similar to the fraction multiplication?

14. The product of three improper fractions is $\frac{14}{3}$. What could the fractions be?

15. Describe a situation at home in which you might multiply $3\frac{1}{2}$ by $2\frac{1}{3}$.

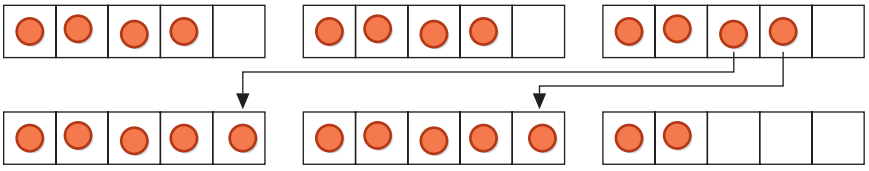
16. Do you agree or disagree with the following statement? Explain. When you multiply a mixed number using thirds by a mixed number using fourths, the answer has to be a mixed number using twelfths.

Mid-Chapter Review

Frequently Asked Questions

Q: **How can you multiply a fraction by a whole number?**

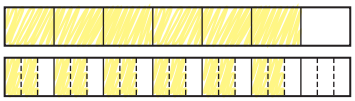
A: You can represent repeated addition using a variety of models. For example, $3 \times \frac{4}{5}$ means 3 sets of $\frac{4}{5}$.



$$3 \times \frac{4}{5} = \frac{4}{5} + \frac{4}{5} + \frac{4}{5} = \frac{12}{5} = 2\frac{2}{5}$$

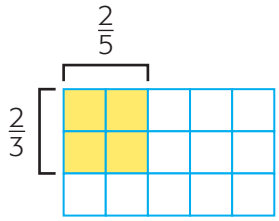
Q: **How can you multiply two fractions less than 1?**

A1: You can model one fraction and then divide it into the appropriate number of pieces. For example, to show $\frac{2}{3} \times \frac{6}{7}$, you can model $\frac{6}{7}$ and divide each of the 6 sevenths into thirds. Then, to show $\frac{2}{3}$ of each section, colour 2 of the thirds.



$$\frac{2}{3} \times \frac{6}{7} = \frac{12}{21}, \text{ or } \frac{4}{7}$$

A2: You can determine the area of a rectangle. For example, to model $\frac{2}{3} \times \frac{2}{5}$, create a rectangle that is $\frac{2}{5}$ of a unit wide and $\frac{2}{3}$ of a unit long and calculate its area. There are 2×2 squares, each with an area of $\frac{1}{15}$. So the total area is $\frac{4}{15}$ square units.



$$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$$

A3: You can multiply the numerators together and the denominators together: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.

For example, $\frac{3}{5} \times \frac{6}{7} = \frac{3 \times 6}{5 \times 7} = \frac{18}{35}$.

Q: How can you estimate the product of two fractions?

A: The product of two fractions close to 1 is close to 1.

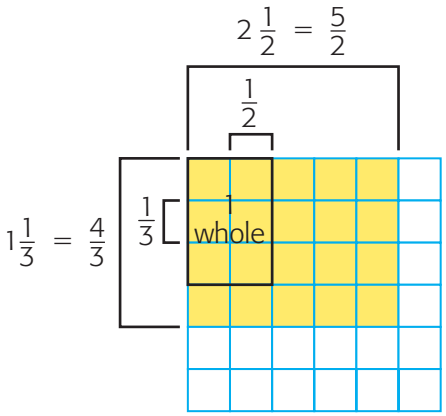
The product of two fractions close to $\frac{1}{2}$ is close to $\frac{1}{4}$. The product of two fractions close to 0 is close to 0.

For example, $\frac{5}{6} \times \frac{7}{8}$ is close to 1 because $\frac{5}{6}$ is close to 1 and $\frac{7}{8}$ is close to one.

Q: How can you multiply two mixed numbers?

A1: You can use an area model to multiply two mixed numbers.

For example, suppose that you wanted to calculate the area of a rectangle that is $1\frac{1}{3}$ units long and $2\frac{1}{2}$ units wide.



There are $4 \times 5 = 20$ squares. Each has an area of $\frac{1}{6}$ of 1 whole.

$\frac{4}{3} \times \frac{5}{2} = \frac{20}{6}$, or $3\frac{2}{6}$, or $3\frac{1}{3}$

A2: You can write each mixed number as an improper fraction. You can multiply the improper fractions like proper fractions.

$$1\frac{1}{3} \times 2\frac{1}{2} = \frac{4}{3} \times \frac{5}{2}$$
$$= \frac{20}{6}$$

Practice

Lesson 2.1

1. Write as a repeated addition. Use fraction strips or a number line to add.
Write each answer as an improper fraction and as a mixed number. Write the fractions in lowest terms.

a) $6 \times \frac{1}{5}$ **c)** $8 \times \frac{3}{5}$

b) $4 \times \frac{5}{12}$ **d)** $5 \times \frac{4}{9}$

2. Use grid paper and counters to multiply.

a) $3 \times \frac{3}{8}$ **c)** $5 \times \frac{5}{6}$

b) $2 \times \frac{5}{9}$ **d)** $4 \times \frac{2}{5}$

3. The product of a fraction and a whole number is $\frac{24}{5}$. What could the fraction and the whole number be?

Lesson 2.2

4. Draw a picture to show $\frac{2}{3}$ of $\frac{3}{8}$.

5. What is the missing fraction?

a) $\frac{1}{4}$ of $\frac{2}{7}$ is ▢. **b)** $\frac{3}{5}$ is ▢ of $\frac{4}{5}$ **c)** ▢ of $\frac{3}{4}$ is $\frac{3}{12}$.

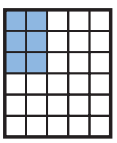
Lesson 2.3

6. Draw a model for each multiplication. Use your model to determine the product.

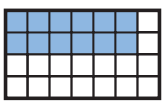
a) $\frac{1}{3} \times \frac{1}{6}$ **b)** $\frac{3}{7} \times \frac{4}{5}$

7. What fraction multiplication does each model represent?

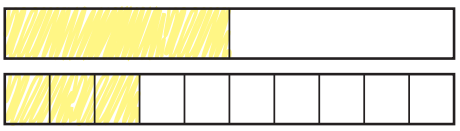
a)



b)



c)



8. If you multiply $\frac{2}{8}$ by another fraction, can the denominator be 20? Explain.

9. About $\frac{3}{4}$ of the traditional dancers of a First Nations school are girls. About $\frac{1}{4}$ of these students are in Grade 8. What fraction of the students who dance are Grade 8 girls?

Lesson 2.4

10. Which products are greater than $\frac{1}{2}$?

a) $\frac{3}{4} \times \frac{5}{6}$

b) $\frac{1}{6} \times \frac{7}{8}$

c) $\frac{3}{9} \times \frac{8}{9}$

d) $\frac{3}{5} \times \frac{2}{3}$

Lesson 2.5

11. Calculate.

a) $\frac{3}{7} \times 3\frac{1}{2}$

b) $\frac{2}{5} \times 1\frac{3}{5}$

c) $1\frac{1}{3} \times 1\frac{2}{3}$

d) $\frac{5}{6} \times 6\frac{12}{25}$

e) $2\frac{3}{4} \times 3\frac{3}{4}$

f) $3\frac{1}{5} \times 6\frac{3}{8}$

12. Eileen used to be on the phone $3\frac{1}{2}$ times as much as her sister every day. As a New Year's resolution, she decided to cut down to about $\frac{2}{5}$ of the time she used to be on the phone. About how many times as much as her sister is Eileen now on the phone?

2.6 Dividing Fractions by Whole Numbers

YOU WILL NEED

- counters
- grids
- Fraction Strips Tower

GOAL

Use a sharing model to represent the quotient of a fraction divided by a whole number.

LEARN ABOUT the Math

Three-tenths of the possible donors still have to be called. Two of the students are going to share the job.

? What fraction of all the possible donors will each student be calling?

A. Suppose $\frac{4}{10}$ of the list of donors still need to be called. Use counters on a grid to represent $\frac{4}{10}$.

B. Arrange the counters into two equal groups.

C. What fraction of the grid is covered by each group?

D. What fraction of the donors will each student phone?

E. How can you change what you did in step B to solve the problem if only $\frac{3}{10}$ of the donors need to be called?

Reflecting

F. What whole number division did you need to do to solve the problem in step D?

G. Why did you need to change your strategy to solve step E?

H. Why is dividing a fraction by 2 the same as multiplying it by $\frac{1}{2}$?

WORK WITH the Math

Example 1 | Relating dividing and multiplying

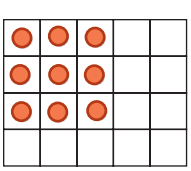
Allison had art class on 9 out of the 20 school days last month. She worked with a partner about $\frac{1}{3}$ of the time. For what fraction of the school days last month did she work with a partner in art?

Allison's Solution

9 out of 20 is $\frac{9}{20}$.

$\frac{9}{20} \div 3$

I wrote 9 out of 20 as a fraction. Then I needed to divide it into 3 equal parts.



$\frac{3}{20}$ are in each group.

$\frac{9}{20} \div 3 = \frac{9 \div 3}{20}$

$= \frac{3}{20}$

I used a grid. I chose a grid with 20 squares to represent 20ths. I made sure my grid had at least 3 rows so I could put the 9 counters into 3 separate rows.

I figured out the fraction in each row.

Example 2 | Relating dividing and multiplying

Allison had art class on 9 out of the 20 school days last month. She worked with a partner about $\frac{1}{3}$ of the time. For what fraction of the school days last month did she work with a partner in art?

Nikita's Solution

$$\frac{9}{20} \div 3 = \frac{1}{3} \times \frac{9}{20}$$
$$= \frac{1 \times 9}{3 \times 20}$$
$$= \frac{9}{60}$$
$$= \frac{9 \div 3}{60 \div 3}$$
$$= \frac{3}{20}$$

I wanted $\frac{1}{3}$ of 9 out of 20.

I wrote 9 out of 20 as $\frac{9}{20}$.

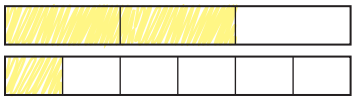
I multiplied by $\frac{1}{3}$ since $\frac{1}{3}$ of something is the same as $\frac{1}{3}$ times that thing.

After I wrote the fraction in lowest terms, to $\frac{3}{20}$, I realized I could have just divided the numerator 9 by 3 and left the denominator as twentieths.

Example 3 | Using a fraction strip model

Divide $\frac{2}{3}$ by 4.

Preston's Solution



$$\frac{2}{3} \div 4 = \frac{1}{6}$$

I used fraction strips to represent $\frac{2}{3}$.

I needed to find strips that were the right length so that four sections made up the $\frac{2}{3}$.

I realized each strip had to be half of $\frac{1}{3}$, which is $\frac{1}{6}$.

A Checking

1. Two-thirds of a room still has to be tiled. Three workers are going to share the job. What fraction of the room will each worker tile if they all work at the same rate?

2. a) Divide $\frac{6}{7}$ by 3 using a grid and counters. Sketch your work.

b) Divide $\frac{5}{7}$ by 3 using a model. Sketch your work.

c) Why might your denominators for parts a) and b) be different?

3. a) How can you solve $\frac{5}{6} \div 4$ using multiplication of fractions?

b) Explain why this works.

B Practising

4. Divide. Use a model for at least two of your solutions and show your work.

a) $\frac{8}{9} \div 4$ **c)** $\frac{6}{9} \div 4$ **e)** $\frac{2}{3} \div 5$

b) $\frac{2}{9} \div 4$ **d)** $\frac{3}{5} \div 6$ **f)** $\frac{7}{8} \div 3$

5. Which quotients are less than $\frac{1}{4}$? How do you know?

a) $\frac{2}{3} \div 3$ **b)** $\frac{7}{8} \div 2$ **c)** $\frac{8}{9} \div 3$

6. Kevin used $\frac{5}{6}$ of a can of paint to cover four walls. How much of a can did he use for each wall?

7. Sheldon used $\frac{1}{6}$ of his blue seed beads to make a Native regalia breastplate. He wanted to use the same colour of beads to make two pairs of moccasins. What fraction of the beads that he originally had could he use for one moccasin?

8. a) Divide $\frac{4}{5}$ by 5.

b) Rewrite $\frac{4}{5}$ as a percent, and divide by 5.

c) Explain how you can use the calculation in part b) to check your answer to part a).

9. a) Create a problem you might solve by dividing $\frac{2}{3}$ by 4.

b) Solve your problem.

10. a) Why do the quotients for $\frac{8}{9} \div 2$, $\frac{8}{12} \div 2$, and $\frac{8}{15} \div 2$ all have the same numerator?

b) Why are the denominators different?

11. Aaron noticed that $\frac{2}{3} \div 5 = \frac{2}{3 \times 5}$, $\frac{4}{5} \div 6 = \frac{4}{5 \times 6}$, and $\frac{3}{5} \div 7 = \frac{3}{5 \times 7}$. What is the pattern he noticed? Is it always true?

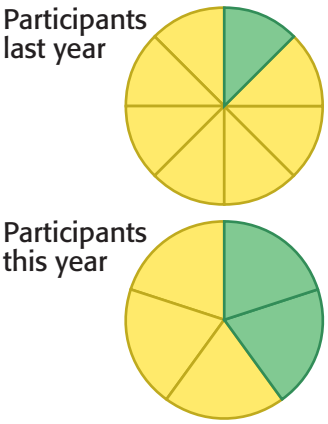
2.7 Estimating Fraction Quotients

YOU WILL NEED
- Fraction Strips Tower

GOAL

Interpret and estimate the quotient of fractions less than 1.

LEARN ABOUT the Math



The fraction of students in a school who participate in school sports has increased from $\frac{1}{8}$ to $\frac{2}{5}$.

? Is the fraction of participating students closer to double or closer to triple what it was?

Example 1 | Comparing fractions by multiplying

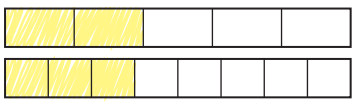
I used fraction strips to compare $\frac{3}{8}$ and $\frac{2}{5}$.

Brian's Solution

$2 \times \frac{1}{8} = \frac{2}{8}$

$\frac{2}{8} < \frac{2}{5}$ since $8 > 5$.

$3 \times \frac{1}{8} = \frac{3}{8}$



Double $\frac{1}{8}$ means $2 \times \frac{1}{8}$.

I wanted to compare $\frac{2}{5}$ to $\frac{2}{8}$. When you compare two fractions with the same numerator, the one with the lower denominator is greater since the whole is divided into fewer parts.

Triple $\frac{1}{8}$ means $3 \times \frac{1}{8}$.

I compared $\frac{3}{8}$ and $\frac{2}{5}$ using fraction strips. They are pretty close in size.

The new fraction is closer to triple $\frac{1}{8}$.

Example 2 | Fitting one fraction into the other fraction

I used fraction strips to visualize the quotient.

Preston's Solution

$\frac{2}{5}$ is about triple $\frac{1}{8}$.

To find out how $\frac{2}{5}$ relates to $\frac{1}{8}$, I need to see how many times $\frac{1}{8}$ fits into $\frac{2}{5}$. That is dividing.

I can see that $\frac{1}{8}$ fits into $\frac{2}{5}$ almost 3 times.

Example 3 | Comparing using equivalents

I used compatible numbers.

Nikita's Solution

$\frac{2}{5} = \frac{2 \times 8}{5 \times 8}$

$= \frac{16}{40}$

$\frac{1}{8} = \frac{1 \times 5}{8 \times 5}$

$= \frac{5}{40}$

$\frac{16}{40}$ is close to $\frac{15}{40}$.

Since $3 \times \frac{5}{40} = \frac{15}{40}$,

$\frac{2}{5}$ is about triple $\frac{1}{8}$.

I want to know, is $\frac{2}{5}$ closer to twice $\frac{1}{8}$ or to three times $\frac{1}{8}$?

I wrote equivalent fractions with the same denominator. Then I used a fraction that was close to one of these and where the numerators divided easily.

Reflecting

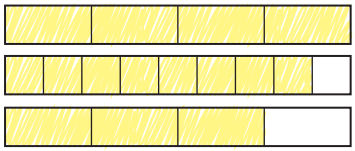
A. Brian and Preston both used fraction strips. Why could Brian solve by multiplying, but Preston solve by dividing?

B. Why did Nikita use equivalent fractions with the same denominator?

Example 4 | Estimating a fraction quotient

To win a recycling contest, the student council knew that at least $\frac{8}{9}$ of the students in the school had to participate. At one point, only $\frac{1}{4}$ of the students had signed up. How many more groups of that size had to sign up to have a chance to win?

Solution A



$\frac{8}{9}$ is closer to 1 than $\frac{3}{4}$.

Almost 4 groups of $\frac{1}{4}$ of the students are needed for the school to have a chance to win.

To determine about how many groups of $\frac{1}{4}$ are in $\frac{8}{9}$, estimate using a whole number of groups.

4 groups is $\frac{4}{4}$.

3 groups is $\frac{3}{4}$.

To decide if $\frac{8}{9}$ is closer to 1 or $\frac{3}{4}$, use a model.

Solution B

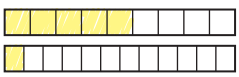
$\frac{8}{9} \div \frac{1}{4}$

is close to

$1 \div \frac{1}{4} = 4$

$\frac{8}{9}$ is close to 1.

There are 4 fourths in 1.



A Checking

1. How does the picture show that $\frac{5}{9} \div \frac{1}{12}$ is about 7?

2. Estimate the quotient as a whole number.

a) $\frac{3}{5} \div \frac{1}{4}$ **b)** $\frac{7}{8} \div \frac{2}{5}$ **c)** $\frac{3}{7} \div \frac{1}{10}$

B Practising

3. How does the picture show that $\frac{5}{6} \div \frac{1}{10}$ is about 8?

4. Draw a picture to estimate about how many times $\frac{2}{5}$ fits into $\frac{8}{9}$.

5. Estimate each quotient as a whole number.

a) $\frac{11}{12} \div \frac{3}{12}$ **c)** $\frac{11}{12} \div \frac{1}{7}$ **e)** $\frac{5}{6} \div \frac{1}{10}$

b) $\frac{11}{12} \div \frac{1}{6}$ **d)** $\frac{3}{4} \div \frac{1}{10}$ **f)** $\frac{10}{11} \div \frac{2}{3}$

6. Amber needs $\frac{3}{4}$ of a cupful of berries to make a Saskatoon berry soup. She can find only a $\frac{1}{3}$-cup measure. About how many times will she have to fill the cup to have the right amount of berries?

7. Why might you estimate $\frac{7}{8} \div \frac{2}{7}$ by dividing 1 by $\frac{1}{4}$?

8. List two fractions you can divide to get the quotient specified.

a) about 2 **b)** a bit more than 3

9. You divide a fraction less than $\frac{1}{2}$ by a fraction less than $\frac{1}{8}$. How could the result be each of the following?

a) close to 4 **b)** close to 8 **c)** close to 20

10. Tom used 25 tiles to tile $\frac{3}{8}$ of the floor. About how many tiles does he need to finish the job? How do you know?

11. Describe a situation that can be answered by estimating $\frac{7}{8} \div \frac{1}{3}$.

12. How do you know that $\frac{3}{4} \div \frac{5}{6}$ is less than 1?

13. **a)** Describe two different ways to estimate $\frac{7}{8} \div \frac{2}{11}$.

b) Which way would you choose? Why?

2.8 Dividing Fractions by Measuring

YOU WILL NEED

- Fraction Strips Tower

GOAL

Divide fractions using models and using equivalent fractions with a common denominator.

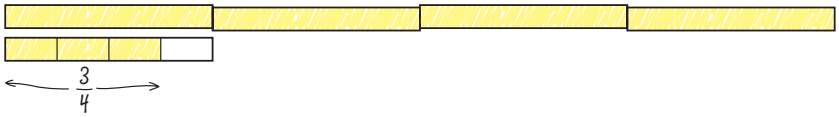
LEARN ABOUT the Math

Misa exercises for $\frac{3}{4}$ of an hour several times a week.

? How many times does Misa have to exercise if she wants to exercise for a total of 4 h every week?

A. Line up 4 whole fraction strips to show a total of 4 ones.

B. Line up enough $\frac{3}{4}$ strips to fit along the four whole strips from step A.



C. Divide each whole strip into 4 fourths.

D. How many times does the $\frac{3}{4}$ strip fit along the 4 whole strips?

E. How many times does Misa have to exercise to achieve her goal of 4 h?

Reflecting

F. Why does finding out how many $\frac{3}{4}$ strips fit along the length of 4 whole strips help you solve the problem?

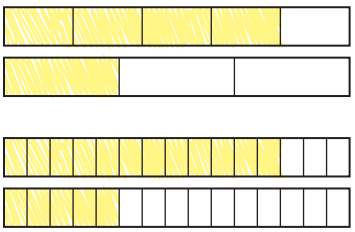
G. How could you solve the problem using equivalent fractions for 4 and $\frac{3}{4}$, and then dividing the numerators?

WORK WITH the Math

Example 1 | Using a model

Calculate $\frac{4}{5} \div \frac{1}{3}$.

Aaron's Solution



To divide $\frac{4}{5}$ by $\frac{1}{3}$, I asked myself how many thirds are in $\frac{4}{5}$.

Since $\frac{1}{3} < \frac{4}{5}$, the answer is more than 1. I lined up fraction strips to estimate.

It looked as if a bit more than 2 thirds fit into $\frac{4}{5}$.

I decided to use equivalent fractions. I chose fifteenths since I was using thirds and fifths.

$$\frac{4}{5} \div \frac{1}{3} = \frac{12}{15} \div \frac{5}{15}$$
$$= \frac{12}{5}$$
$$= 2\frac{2}{5}$$

I counted how many times $\frac{5}{15}$ fits into $\frac{12}{15}$.

I realized that the quotient was just $12 \div 5$. Once the denominators are equal, you only have to divide the numerators.

Example 2 | Using common denominators

Calculate $\frac{1}{3} \div \frac{2}{5}$.

Allison's Solution

$$\frac{1}{3} \div \frac{2}{5} = \frac{1 \times 5}{3 \times 5} \div \frac{2 \times 3}{5 \times 3}$$
$$= \frac{5}{15} \div \frac{6}{15}$$
$$= \frac{5}{6}$$

To calculate $\frac{1}{3} \div \frac{2}{5}$, I wanted to find out how many times $\frac{2}{5}$ fits into $\frac{1}{3}$. I cannot fit an entire $\frac{2}{5}$ into $\frac{1}{3}$, so the answer must be less than 1. But I can fit most of $\frac{2}{5}$ into $\frac{1}{3}$, so the answer should be close to 1. I solved the problem using a common denominator.

A common denominator for $\frac{1}{3}$ and $\frac{2}{5}$ is $3 \times 5 = 15$.

I divided the numerators to determine how many $\frac{6}{15}$ are in $\frac{5}{15}$. The answer makes sense. It is less than 1, but close to 1.

Example 3 | Dividing a mixed number by a fraction

There were $2\frac{1}{2}$ containers of orange juice in Jeff's fridge. How many glasses of juice can he pour if each glass uses about $\frac{1}{5}$ of a container?

Misa's Solution

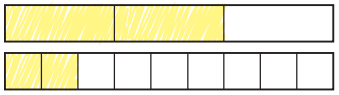
$$2\frac{1}{2} = \frac{(2 \times 2) + 1}{2}$$
$$= \frac{5}{2}$$
$$\frac{5}{2} \div \frac{1}{5} = \frac{25}{10} \div \frac{2}{10}$$
$$= 25 \div 2$$
$$= 12\frac{1}{2}$$

I needed to divide $2\frac{1}{2}$ by $\frac{1}{5}$ to figure out how many glasses Jeff can pour.

I renamed $2\frac{1}{2}$ as an improper fraction.

Then I divided by $\frac{1}{5}$ using a common denominator of 2×5.

I just had to divide the numerators.



A Checking

1. What division expression does this picture represent?

2. Draw a fraction strip model to show the number of times $\frac{1}{4}$ fits into $\frac{7}{8}$.

3. Calculate.
   a) $\frac{5}{8} \div \frac{3}{4}$ **b)** $2\frac{1}{2} \div \frac{2}{3}$

4. Craig needs to measure $3\frac{1}{3}$ cups. How many times must he fill a $\frac{1}{2}$-cup measure?

B Practising

5. What division expression does each picture represent?
   a)
   b)

6. Calculate each quotient using equivalent fractions. Explain your thinking for part d).
   a) $5 \div \frac{1}{3}$ **c)** $2\frac{1}{2} \div \frac{3}{8}$
   b) $1\frac{3}{4} \div \frac{5}{6}$ **d)** $\frac{3}{5} \div \frac{5}{6}$

7. Frederika has written $\frac{2}{5}$ of a page for her report in 1 h. About how much time will she need to complete the entire report at this rate?

8. Create and solve a problem that can be solved by dividing $\frac{1}{3}$ by $\frac{1}{10}$.

9. Alana is cooking a turkey. It takes $4\frac{1}{2}$ h to cook. She checks it every 20 min, or $\frac{1}{3}$ of an hour.
   a) How many times will she check the turkey before it is cooked?
   b) Why can you keep subtracting $\frac{1}{3}$ from $4\frac{1}{2}$ to answer the question?

10. How can you calculate $\frac{3}{5} \div \frac{1}{2}$ using equivalent fractions with a common denominator?

11. Calculate. Write fractions in lowest terms.

a) $\frac{1}{4} \div \frac{4}{6}$ **c)** $\frac{4}{9} \div \frac{3}{4}$ **e)** $2\frac{1}{4} \div 1\frac{5}{6}$

b) $\frac{3}{5} \div \frac{2}{3}$ **d)** $2\frac{1}{3} \div 1\frac{7}{8}$ **f)** $1\frac{7}{9} \div 2\frac{1}{3}$

12. Craig has only a $\frac{1}{3}$-cup measuring cup. What operation would you perform to answer each question?

a) How much flour could Craig measure by filling the cup 5 times?

b) How many times would Craig have to fill his measuring cup to measure $2\frac{3}{8}$ cups of flour?

13. Does order matter when you divide fractions? For example, is $\frac{2}{3} \div \frac{1}{5}$ the same as $\frac{1}{5} \div \frac{2}{3}$? Explain.

14. How do you know that dividing by $\frac{1}{6}$ is the same as multiplying by 6?

15. Teo made a video that was $2\frac{1}{2}$ h long. He made it by clipping together sections that were each about $\frac{1}{3}$ of an hour long.

a) What operation could you perform to decide about how many sections Teo clipped together?

b) How do you know that the sections were not all exactly $\frac{1}{3}$ of an hour long?

16. How would you explain to someone why $\frac{5}{a} \div \frac{2}{a} = 2\frac{1}{2}$, no matter what the denominator is?

It Is Just Like Multiplying!

Did you know that you can divide fractions by dividing numerators and dividing denominators?

For example,

$$\frac{15}{16} \div \frac{3}{4} = \frac{15 \div 3}{16 \div 4}$$
$$= \frac{5}{4}$$

It is just like multiplying numerators and denominators to multiply fractions.

1. How do you know that $\frac{15}{16} \div \frac{3}{4}$ really is $\frac{5}{4}$?
2. How does using the equivalent fraction $\frac{9}{15}$ help you use this "dividing numerators/dividing denominators" method to calculate $\frac{3}{5} \div \frac{1}{3}$?
3. How could you use equivalent fractions to calculate $\frac{2}{3} \div \frac{3}{4}$ using this method?
4. Why do you think this method works?
5. When would you be most likely to use this method?

2.9 Dividing Fractions Using a Related Multiplication

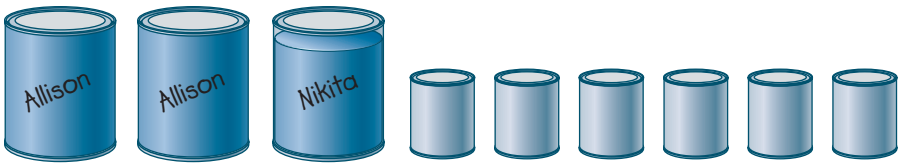
YOU WILL NEED
- Fraction Strips Tower

GOAL

Divide fractions using a related multiplication.

LEARN ABOUT the Math

Allison has 2 large cans of paint. Nikita has $\frac{7}{8}$ of a large can of paint. Each student is pouring paint into small cans that hold $\frac{1}{3}$ as much as the large ones.



? How many small cans of paint will each student fill?

Example 1 | Dividing a whole number by a fraction

To find out how many small cans I can fill, I divided 2 by $\frac{1}{3}$.

Allison's Solution

$2 \div \frac{1}{3} = 2 \times 3$
$= 6$

My 2 large cans of paint will fill 6 small cans.

I needed to divide 2 by $\frac{1}{3}$ to see how many $\frac{1}{3}$-size cans would be filled by 2 large cans.

The small can is $\frac{1}{3}$ the size, so each large can fills 3 small ones. I double that for 2 large cans.

Example 2 | Dividing a fraction by a fraction

Calculate $\frac{7}{8} \div \frac{1}{3}$.

Nikita's Solution

$1 \div \frac{1}{3} = 3$

$\frac{7}{8} \div \frac{1}{3} = \frac{7}{8} \times 3$

$= \frac{21}{8}$

$= 2\frac{5}{8}$

My $\frac{7}{8}$-full large can will fill $2\frac{5}{8}$ small cans.

I used a related multiplication to divide $\frac{7}{8}$ by $\frac{1}{3}$.

I needed to divide $\frac{7}{8}$ by $\frac{1}{3}$ to see how many $\frac{1}{3}$-size cans fit into $\frac{7}{8}$ of a large can.

I realized it would have to be $\frac{7}{8}$ as much as the amount that 1 whole large can of paint fills. Since 1 large can fills 3 small ones, I multiplied $\frac{7}{8}$ by 3.

It makes sense that the answer is less than 3, but close to it.

reciprocal

the fraction that results from switching the numerator and denominator; for example, $\frac{4}{5}$ is the reciprocal of $\frac{5}{4}$

Reflecting

A. Why did Allison and Nikita divide by $\frac{1}{3}$ to solve the problem?

B. The result when Allison divided by $\frac{1}{3}$ was twice as much as the reciprocal of $\frac{1}{3}$. Why does that make sense?

C. Suppose the small can had held $\frac{2}{3}$ as much as the large can instead of only $\frac{1}{3}$ as much. Why could Allison and Nikita have multiplied both 2 and $\frac{7}{8}$ by the reciprocal of $\frac{2}{3}$?

Example 3 | Dividing a mixed number by a fraction

Misa wants to pour $1\frac{7}{8}$ large cans of paint into small cans. Each small can holds $\frac{3}{5}$ as much paint as a large can. How many small cans will Misa fill?

Solution A: Using fraction strips to divide

Estimate:

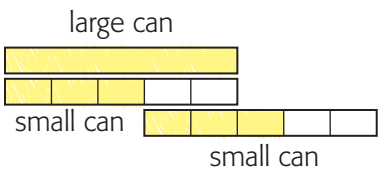
$\frac{3}{5}$ is about $\frac{1}{2}$

$1\frac{7}{8}$ is about 2

There are 4 halves in 2.

Number of small cans in 1 large can.

$1 \div \frac{3}{5} = \frac{5}{3}$, or $1\frac{2}{3}$ small cans



$1\frac{2}{3} \times 1\frac{7}{8}$

$= \frac{5}{3} \times \frac{15}{8}$

$= \frac{75}{24}$, or $\frac{25}{8}$, or $3\frac{1}{8}$

Misa will fill $3\frac{1}{8}$ small cans with paint.

Estimate first.

It takes about 2 small cans to fill one large one.

Since there are almost 2 large cans, Misa will need about 4 cans.

Calculate next.

You can use fraction strips to see how many times $\frac{3}{5}$ fits into 1.

The fraction part is $\frac{2}{3}$ since, even though it is $\frac{2}{5}$ of a can, it is $\frac{2}{3}$ of a small can.

Then figure out how many times $\frac{3}{5}$ fits in $1\frac{7}{8}$ cans by multiplying the number for 1 large can by $1\frac{7}{8}$.

Solution B: Using a common denominator

$$1\frac{7}{8} \div \frac{3}{5} = \frac{15}{8} \div \frac{3}{5}$$
$$= \frac{15 \times 5}{8 \times 5} \div \frac{3 \times 8}{5 \times 8}$$
$$= \frac{75}{40} \div \frac{24}{40}$$
$$= \frac{75}{24}, \text{ or } 3\frac{3}{24}, \text{ or } 3\frac{1}{8}$$

Misa will fill $3\frac{1}{8}$ small cans with paint.

Solution C: Multiplying by the reciprocal

$$1\frac{7}{8} \div \frac{3}{5} = \frac{15}{8} \div \frac{3}{5}$$
$$= \frac{15}{8} \times \frac{5}{3}$$
$$= \frac{75}{24}, \text{ or } 3\frac{3}{24}, \text{ or } 3\frac{1}{8}$$

Misa will fill $3\frac{1}{8}$ small cans with paint.

A Checking

1. Calculate.

a) $\frac{3}{8} \div \frac{1}{2}$ **b)** $\frac{7}{8} \div \frac{1}{3}$

2. Lynnsie has $1\frac{1}{2}$ large cans of paint. Each small can holds $\frac{3}{5}$ as much paint as a large can. How many small cans can Lynnsie fill?

B Practising

3. Calculate.

a) $\frac{3}{9} \div \frac{2}{9}$ **c)** $\frac{4}{8} \div \frac{7}{8}$ **e)** $\frac{1}{5} \div \frac{2}{5}$

b) $\frac{1}{2} \div \frac{1}{3}$ **d)** $\frac{4}{5} \div \frac{2}{3}$ **f)** $\frac{9}{20} \div \frac{3}{5}$

4. Rahul has $\frac{2}{3}$ of a container of trail mix. He is filling snack packs that each use about $\frac{1}{5}$ of a container. How many snack packs can Rahul make?

5. Why does it make sense that $\frac{7}{8} \div \frac{3}{4}$ is greater than $\frac{7}{8}$?

6. Which quotients are $1\frac{1}{4}$? How do you know?

a) $\frac{5}{2} \div \frac{1}{2}$ **c)** $\frac{3}{5} \div \frac{3}{4}$

b) $\frac{3}{4} \div \frac{3}{5}$ **d)** $5 \div 4$

7. Estimate each quotient. Then express the mixed numbers as improper fractions and calculate the exact quotient.

a) $6\frac{2}{5} \div 3\frac{1}{2}$ **b)** $5\frac{2}{3} \div 1\frac{1}{4}$ **c)** $8\frac{3}{4} \div 2\frac{1}{3}$

8. Calculate.

a) $\frac{9}{8} \div \frac{3}{8}$ **c)** $\frac{11}{4} \div \frac{3}{2}$ **e)** $5\frac{1}{3} \div 2\frac{3}{4}$

b) $\frac{7}{3} \div \frac{5}{6}$ **d)** $1\frac{2}{3} \div \frac{3}{7}$ **f)** $6\frac{3}{5} \div 2\frac{1}{3}$

9. a) Which quotients are greater than 1?

i) $\frac{3}{5} \div \frac{2}{3}$ **ii)** $\frac{9}{2} \div \frac{5}{6}$ **iii)** $\frac{3}{7} \div \frac{1}{8}$

b) How could you have predicted the answers to part a) without calculating the quotients?

10. Which quotients are greater than 2? Calculate these quotients only.

a) $\frac{5}{9} \div \frac{1}{4}$ **b)** $3\frac{1}{3} \div \frac{4}{5}$ **c)** $\frac{8}{9} \div \frac{3}{4}$ **d)** $\frac{7}{8} \div \frac{1}{3}$

11. Choose two fractions where the quotient is less than the product.

12. Printers print at different rates. How many pages does each printer print each minute?

a) 20 pages in $1\frac{1}{2}$ min

b) 20 pages in $1\frac{1}{3}$ min

13. Miri filled $2\frac{1}{2}$ pitchers with $\frac{2}{3}$ of the punch she made. How many pitchers would she fill if she used all the punch she made?

14. Trevor takes $4\frac{1}{2}$ min to run once around his favourite route. How many laps can he do in each time period?

a) 30 min **b)** 20 min **c)** 15 min

15. a) Calculate $0.45 \div 0.3$ using decimal division.

b) Calculate $\frac{45}{100} \div \frac{3}{10}$ using fraction division.

c) What do you notice?

16. A pattern block design is made up of the equivalent of $3\frac{1}{3}$ red blocks. How many blue blocks could cover that design?

17. Describe a situation in which you might use each calculation.

a) $\frac{9}{8} \div \frac{2}{3}$ **b)** $1\frac{2}{5} \div 2\frac{2}{3}$

Math GAME

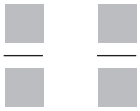
YOU WILL NEED
- a pair of dice

Target $\frac{2}{3}$

In this game, you will roll a pair of dice twice to create two fractions. Then you will add, subtract, multiply, or divide your fractions to get an answer as close as possible to $\frac{2}{3}$.

Number of players: 2, 3, or 4

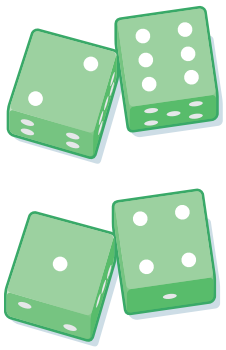
How to Play



1. Each player rolls the pair of dice twice, then uses the four numbers as the numerators and denominators of two fractions.
2. Each player can add, subtract, multiply, or divide the two fractions to get an answer as close as possible to $\frac{2}{3}$.
3. The player with the answer closest to $\frac{2}{3}$ gets a point. Both players get a point if there is a tie.
4. Keep playing until one player has 10 points.

Preston's Turn

I rolled a 2 and a 6, then I rolled a 1 and a 4.



+	−	×	÷
$\frac{2}{4}+\frac{1}{6}$	$\frac{4}{6}-\frac{1}{2}$	$\frac{1}{2}\times\frac{4}{6}$	$\frac{1}{6}\div\frac{2}{4}$
$=\frac{6}{12}+\frac{2}{12}$	$=\frac{4}{6}-\frac{3}{6}$	$=\frac{4}{12}$	$=\frac{1}{6}\times\frac{4}{2}$
$=\frac{8}{12}$	$=\frac{1}{6}$	$=\frac{1}{3}$	$=\frac{4}{12}$
$=\frac{2}{3}$			$=\frac{1}{3}$

I could use $\frac{2}{4}+\frac{1}{6}$ to get exactly $\frac{2}{3}$.

2.10 Order of Operations

YOU WILL NEED

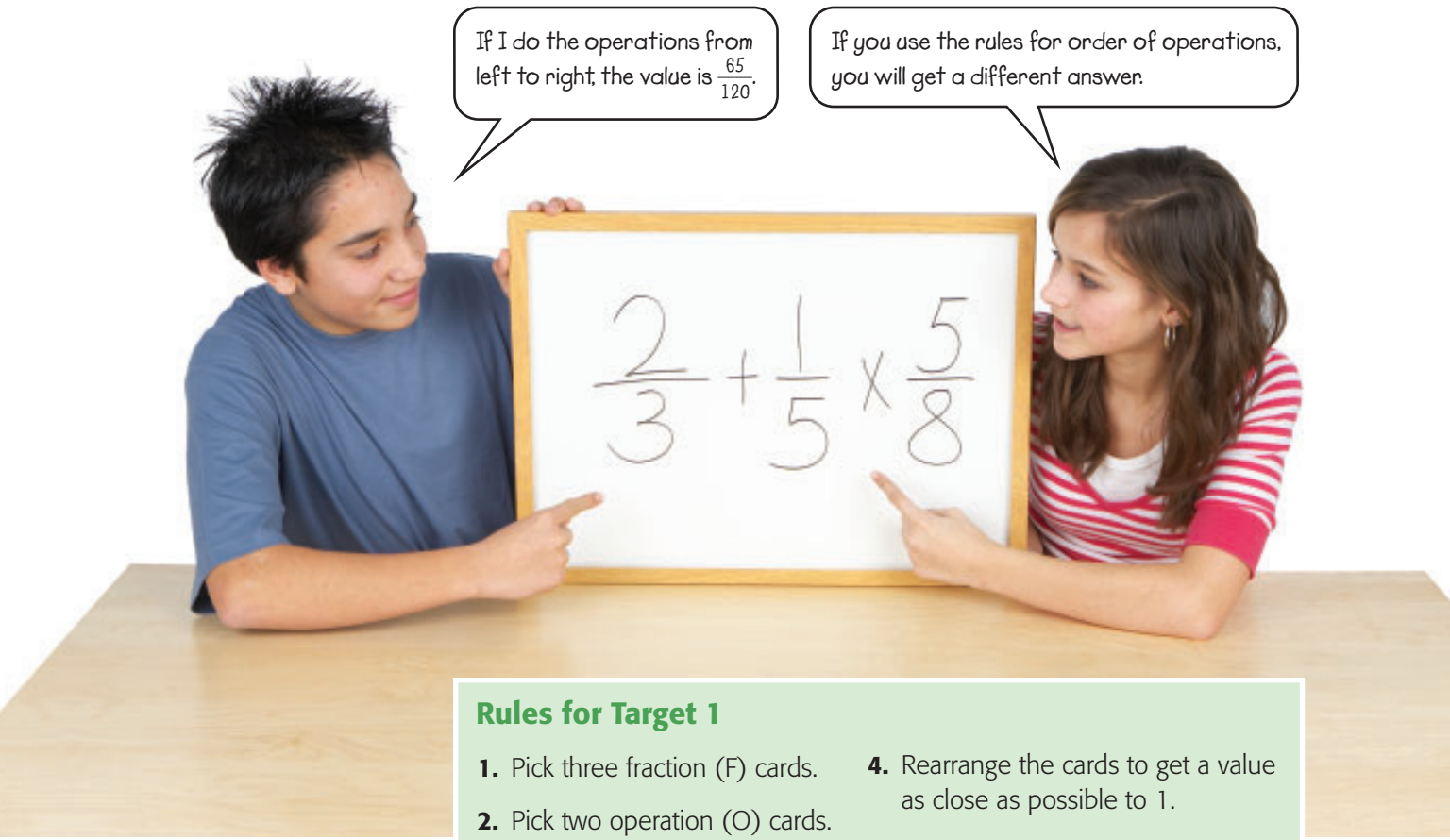
- Fractions and Operations Cards I
- Fractions and Operations Cards II

GOAL

Use the order of operations in calculations involving fractions.

LEARN ABOUT the Math

Allison and Preston are playing a math game called Target 1.



Rules for Target 1

1. Pick three fraction (F) cards.
2. Pick two operation (O) cards.
3. Put them in this order: F O F O F
4. Rearrange the cards to get a value as close as possible to 1.
5. The closest value gets 1 point.
6. The first player to get 5 points wins.

How close to 1 can Allison get with her cards?

A. Show how Preston got $\frac{65}{120}$.

B. What would Allison's answer be if she were to use the rules for the order of operations? Can she get any closer to 1 by rearranging her cards?

Reflecting

C. How could you use brackets to get the same answer as Preston in step A?

D. What is the correct order of operations for Allison's original calculation?

WORK WITH the Math

Example 1 | Using the order of operations with fractions

Calculate $\frac{7}{3} - \frac{4}{5} \times \left(\frac{5}{6} \div \frac{1}{2}\right) + \frac{1}{4}$

Preston's Solution

$\frac{7}{3} - \frac{4}{5} \times \left(\frac{5}{6} \div \frac{1}{2}\right) + \frac{1}{4}$
$= \frac{7}{3} - \frac{4}{5} \times \frac{10}{6} + \frac{1}{4}$

First I had to do $\frac{5}{6} \div \frac{1}{2}$ since it was in brackets.

I used the reciprocal to calculate $\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1}$ using mental math.

$= \frac{7}{3} - \left(\frac{4}{5} \times \frac{10}{6}\right) + \frac{1}{4}$
$= \frac{7}{3} - \frac{40}{30} + \frac{1}{4}$

You are supposed to do multiplications before additions.

I added brackets to show the multiplication I would do next.

$= \left(\frac{7}{3} - \frac{4}{3}\right) + \frac{1}{4}$
$= \frac{3}{3} + \frac{1}{4}$
$= 1\frac{1}{4}$

I rewrote the product $\frac{40}{30}$ in lower terms. Then, I subtracted and added from left to right.

I added brackets to show my thinking.

A Checking

1. Calculate using the rules for order of operations.

a) $3 + \frac{1}{2} \div \frac{2}{3} \times 8$ **b)** $\frac{2}{3} + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

2. Suppose that Nikita picked these cards in the game Target 1:

$\frac{1}{4}$	$\frac{2}{5}$	$\frac{3}{8}$	$\div$	$+$

List three different ways that she could arrange the cards. Then calculate the value for each arrangement.

B Practising

3. Calculate using the rules for order of operations.

a) $\frac{3}{4} + \frac{1}{2} \times \frac{2}{3}$

b) $\frac{5}{6} \div 5 + 3 \times \frac{1}{2}$

c) $\frac{1}{2} - \frac{1}{3} \times \frac{1}{4} + \frac{1}{5} \div \frac{1}{6}$

d) $\left(\frac{1}{2} - \frac{1}{3}\right) \times \left(\frac{1}{4} + \frac{1}{5} \div \frac{1}{6}\right)$

e) $\left(\frac{1}{2} - \frac{1}{3} \times \frac{1}{4} + \frac{1}{5}\right) \div \frac{1}{6}$

f) $\frac{5}{4} \times \frac{1}{2} - \frac{2}{3} \div 2 + \frac{1}{2}$

4. Suppose that Nikita picked these cards in Target 1:

$\frac{1}{2}$	$\frac{4}{5}$	$\frac{2}{9}$	$\div$	$-$

a) List three values greater than 0 that she could calculate, without using brackets, by placing the cards in different positions.

b) Show another value she could calculate if she were allowed to use brackets.

5. Which expressions have the same value?

a) $\frac{2}{3} \div \frac{5}{7} \times \frac{3}{6} + \frac{1}{2}$

b) $\frac{2}{3} \div \left(\frac{5}{7} \div \frac{3}{6}\right) + \frac{1}{2}$

c) $\frac{2}{3} \div \frac{5}{7} \times \left(\frac{3}{6} + \frac{1}{2}\right)$

d) $\frac{2}{3} \div \left(\frac{5}{7} \times \frac{3}{6} + \frac{1}{2}\right)$

6. Calculate.

a) $\frac{6}{7} - \frac{3}{4} \times \left(\frac{3}{5} + \frac{2}{10}\right)$

b) $\frac{5}{8} \div \frac{1}{10} + \frac{1}{3}$

c) $\frac{5}{4} + 2\frac{1}{2} \times 3 \div \frac{2}{3}$

d) $\frac{3}{7} \div \frac{4}{5} + \frac{1}{4}$

e) $\frac{4}{9} + \frac{2}{3} \times \frac{4}{5} \div \frac{1}{10}$

f) $\frac{8}{9} \times \left(\frac{2}{5} + \frac{3}{7} \times \frac{1}{3} + \frac{3}{5}\right) \times 4$

7. a) Calculate each.

i) $\frac{2}{3} + \left(\frac{1}{4} \times \frac{4}{5}\right) - \frac{1}{10}$

ii) $\frac{2}{3} + \frac{1}{4} \times \left(\frac{4}{5} - \frac{1}{10}\right)$

iii) $\left(\frac{2}{3} + \frac{1}{4}\right) \times \frac{4}{5} - \frac{1}{10}$

b) How do your results in part a) show the importance of using brackets in mathematical expressions?

8. What is the missing digit in the following equation?

$\frac{5}{\blacksquare} + \left(\frac{3}{4} - \frac{2}{3}\right) \times \left(\frac{3}{4} - \frac{2}{3}\right) = \frac{1}{24}$

9. Use two pairs of brackets to make the following equation true.

$2 + \frac{1}{4} + \frac{1}{3} \times \frac{3}{7} - \frac{2}{5} \times \frac{3}{8} \div \frac{1}{10} + \frac{1}{5} = 1\frac{3}{4}$

10. What values of a, b, and c will make the value of the expression below greater than $1\frac{1}{2}$? Determine two sets of possibilities using only proper fractions.

$a - b + c \times c$

11. Which expressions have values less than 1?

a) $\frac{1}{2} + \left(1 - \frac{3}{4}\right) \times \frac{1}{2}$

b) $2 \div \frac{7}{10} \times \frac{1}{3}$

c) $\frac{7}{8} \times 1\frac{1}{4} \div \left(\frac{3}{16} - \frac{1}{8}\right)$

d) $2\frac{2}{3} \times \frac{1}{5} + \frac{1}{5}$

12. Create an expression involving fractions and operation signs that results in a whole number only if the correct order of operations is used.

13. How does knowing the order of operations help make sure that you get the same answer to $\frac{2}{3} + \frac{1}{3} \times \frac{1}{12}$ as other students in the class?

2.11 Communicate about Multiplication and Division

GOAL

Describe situations involving multiplying and dividing fractions and mixed numbers.

LEARN ABOUT the Math

Preston created a problem that could be solved using this calculation: $3 + \frac{2}{3} \times 3\frac{1}{2}$

A cookie recipe used $3\frac{1}{2}$ cups of sugar and $4\frac{1}{2}$ cups of flour. Preston had only 3 cups of flour so he used $\frac{2}{3}$ of $3\frac{1}{2}$ cups of sugar. Altogether, how much flour and sugar did he use?

He was trying to explain why the problem was a correct one to use.

Preston's Explanation

The problem has an adding part and a multiplying part.

The adding part has to be about combining things, and the multiplying part has to be about taking part of something.

Since I needed $3\frac{1}{2}$ of something, I decided to use cups in a recipe.

I decided the problem would involve taking $\frac{2}{3}$ of the $3\frac{1}{2}$ and adding it to the 3 that was already there.

The 3 also had to be cups. I made the recipe start with $4\frac{1}{2}$ cups of flour, because I know 3 is $\frac{2}{3}$ of $4\frac{1}{2}$.

Aaron's Questions

Why does the multiplying have to be about taking part of something?

Why did you take $\frac{2}{3}$ of $3\frac{1}{2}$ instead of $3\frac{1}{2}$ of $\frac{2}{3}$?

Why did 3 already have to be there?

How can you improve Preston's explanation?

A. How can you respond to Aaron's questions to improve Preston's explanation?

B. What other questions could Aaron have asked?

Communication Checklist

- ✔ Did you explain each step?
- ✔ Did you justify your conclusions?
- ✔ Did you use models to make your thinking clear?

Reflecting

C. Which parts of the Communication Checklist did Preston cover well? Which parts did Aaron cover in his questions?

D. How would you modify Preston's explanation to explain why his problem is appropriate?

WORK WITH the Math

Example 1 | Describing a situation for dividing fractions

Create a problem that requires division of $1\frac{1}{2}$ by $\frac{4}{5}$.

a) Explain why the problem requires that division.

b) Explain how and why the problem could also be solved using multiplication.

Misa's Solution

a) Jeff's mom was installing new baseboards in a room. She had a lot of strips of wood. Most were one length, and there were a few shorter ones that were $\frac{4}{5}$ of that length.

She had to fill a space that required $1\frac{1}{2}$ of the longer strips. If she decided to use the shorter strips, how many of them would she need?

I know that one meaning of division is how many of one thing fit into another. I decided to use that meaning. I picked a problem about strips of wood.

I made sure one strip was $\frac{4}{5}$ as long as a certain distance and the other strip was $1\frac{1}{2}$ times as long as that same distance.

b) $1\frac{1}{2} \div \frac{4}{5} = \frac{3}{2} \div \frac{4}{5}$

$= \frac{3}{2} \times \frac{5}{4}$

$= \frac{15}{8}$

I know that one way to solve a division question involving fractions is to multiply by the reciprocal. So to solve the problem I created, I could use multiplication of fractions.

A Checking

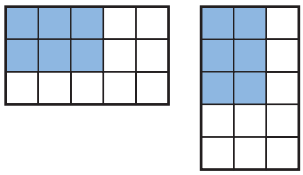
1. a) Create a problem that requires multiplying $\frac{2}{3} \times \frac{8}{9}$ and solve it.

b) Explain why multiplying these numbers is appropriate for the problem.

c) How do you know your answer is reasonable?

B Practising

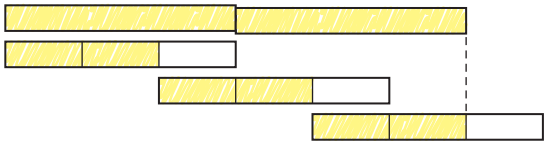
2. Use words and these grids to explain why $\frac{3}{5}$ of $\frac{2}{3}$ is the same as $\frac{2}{3}$ of $\frac{3}{5}$.



3. a) Create a problem that requires calculating $\frac{3}{5} \div 4$ as well as a multiplication by 3.

b) Explain why a different computation could solve that problem.

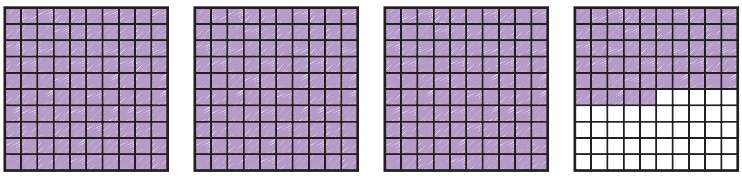
4. Explain why you can calculate $2 \div \frac{2}{3}$ using each method below. Use the Communication Checklist and the picture.



a) Divide equivalent fractions with the same denominator.

b) Multiply 2 by 3 and then divide by 2.

5. Complete Diane's explanation for calculating 1.2×3.55.



1.2 is $1\frac{2}{10}$. This is the same as $1\frac{1}{5}$. So I need 3.55 and another $\frac{1}{5}$ of 3.55.

6. How can you use fraction multiplication to explain why $4.2 \times 0.2 = 0.84$?

7. **a)** Why can you calculate 60% of 1.5 by multiplying $\frac{3}{5} \times \frac{3}{2}$?

b) Do you think this is the easiest way to calculate the percent? Explain.

8. Fabienne said that she now understands why she needs to multiply the numerator and denominator of a fraction by the same amount to get an equivalent fraction. Explain her reasoning, shown below.

$$\frac{3}{5} \times 1 = \frac{3}{5}$$
$$1 = \frac{2}{2}$$
$$\frac{3}{5} \times \frac{2}{2} = \frac{3}{5}$$
$$\frac{3 \times 2}{5 \times 2} = \frac{3}{5}$$

9. Robin said that, when he multiplies or divides mixed numbers, he usually uses decimal equivalents instead because it is easier. Do you agree or disagree? Explain using examples.

10. Explain how multiplying fractions is like multiplying whole numbers and how it is different.

11. Explain how you know that $3\frac{1}{2} \times 6\frac{1}{3}$ must be greater than 21 before you do the calculation.

12. Explain why $\frac{15}{16} \div \frac{5}{8}$ is half of $\frac{15}{16} \div \frac{5}{16}$.

Chapter Self-Test

1. Draw a model to show that $4 \times \frac{2}{3} = \frac{8}{3}$.

2. Use fraction strips to model each.

a) $\frac{2}{3}$ of $\frac{1}{4}$ is ■. **b)** $\frac{1}{2}$ of $\frac{6}{9}$ is ■. **c)** $1\frac{1}{5}$ of $\frac{4}{7}$ is ■.

3. Explain why multiplying a fraction by $\frac{5}{6}$ results in a value that is less than the original fraction.

4. Calculate.

a) $\frac{3}{5} \times \frac{5}{8}$ **b)** $\frac{2}{6} \times \frac{5}{6}$ **c)** $\frac{3}{7} \times \frac{1}{6}$ **d)** $\frac{5}{8} \times \frac{3}{7}$

5. Show two ways to calculate $1\frac{2}{3} \times 2\frac{1}{4}$.

6. Calculate.

a) $\frac{3}{5} \times 5\frac{1}{2}$ **c)** $1\frac{2}{5} \times 1\frac{3}{4}$

b) $\frac{7}{10} \times 6\frac{3}{4}$ **d)** $\frac{2}{9} \times 5\frac{1}{3}$

7. **a)** Draw a picture to show that $\frac{3}{4} \div \frac{5}{8}$ is $1\frac{1}{5}$.

b) Use a multiplication equation to show that $\frac{3}{4} \div \frac{5}{8} = 1\frac{1}{5}$.

8. Calculate.

a) $\frac{4}{5} \div \frac{2}{5}$ **b)** $\frac{5}{8} \div \frac{1}{5}$ **c)** $\frac{1}{5} \div \frac{5}{8}$ **d)** $3\frac{1}{2} \div 1\frac{1}{4}$

9. Calculate.

a) $\frac{2}{3} - \frac{1}{4} \times \frac{5}{6} \div 2$

b) $\left(\frac{2}{3} - \frac{1}{4}\right) \times \left(\frac{5}{6} \div 2\right)$

c) $\frac{2}{3} - \left(\frac{1}{4} + \frac{9}{12}\right) \times \frac{5}{2} \times \frac{5}{6} \div 5$

What Do You Think Now?

Revisit What Do You Think? on page 45. How have your answers and explanations changed?

Chapter Review

Frequently Asked Questions

Q: **How can you divide a fraction by a whole number?**

A1: You can think of it as sharing. For example, $\frac{4}{5} \div 3$ tells you the share size if 3 people share $\frac{4}{5}$ of something.

A2: You can use a model.

$$\frac{4}{5} \div 3 = \frac{4}{15}$$

A3: You can multiply by a fraction.

$$\frac{4}{5} \div 3 = \frac{4}{5} \times \frac{1}{3}$$
$$= \frac{4}{15}$$

A4: You can divide using an equivalent fraction where the numerator is a multiple of the whole number.

$$\frac{4}{5} \div 3 = \frac{12}{15} \div 3$$
$$= \frac{4}{15}$$

Q: **How can you divide two fractions?**

A1: You can determine the number of times the divisor fits into the dividend using fraction strips and a common denominator.

$$\frac{2}{3} \div \frac{4}{9} = \frac{6}{9} \div \frac{4}{9}$$
$$= 6 \div 4$$
$$= 1\frac{2}{4} \text{ or } 1\frac{1}{2}$$

A2: You can multiply by the reciprocal. For example,

$$\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \times \frac{9}{4}$$
$$= \frac{18}{12}$$
$$= 1\frac{6}{12} \text{ or } 1\frac{1}{2}$$

Q: In what order do you perform a series of fraction calculations?

A: Use the rules for the order of operations:

Perform the operations in brackets first.

Next, divide and multiply from left to right.

Then, add and subtract from left to right.

Then, convert the final answer to a mixed number.

For example,

$$\frac{3}{2} - \frac{2}{5} \div \frac{1}{5} \times \frac{3}{10} + \frac{2}{3}$$
$$= \frac{3}{2} - \left(\frac{2}{5} \div \frac{1}{5}\right) \times \frac{3}{10} + \frac{2}{3}$$
$$= \frac{3}{2} - 2 \times \frac{3}{10} + \frac{2}{3}$$
$$= \frac{3}{2} - \left(2 \times \frac{3}{10}\right) + \frac{2}{3}$$
$$= \frac{3}{2} - \frac{6}{10} + \frac{2}{3}$$
$$= \left(\frac{3}{2} - \frac{6}{10}\right) + \frac{2}{3}$$
$$= \left(\frac{15}{10} - \frac{6}{10}\right) + \frac{2}{3}$$
$$= \frac{9}{10} + \frac{2}{3}$$
$$= \frac{27}{30} + \frac{20}{30}$$
$$= \frac{47}{30}, \text{ or } 1\frac{17}{30}$$

Practice

Lesson 2.1

1. Draw a model to represent $3 \times \frac{5}{6}$.

2. Calculate each. Express the answer as a whole or mixed number.

a) $8 \times \frac{4}{5}$ **b)** $6 \times \frac{3}{5}$ **c)** $9 \times \frac{2}{7}$ **d)** $12 \times \frac{2}{3}$

Lesson 2.2

3. What is the value of each expression?

a) $\frac{1}{5}$ of $\frac{1}{2}$ **b)** $\frac{3}{8}$ of $\frac{8}{9}$ **c)** $\frac{2}{3}$ of $\frac{6}{8}$ **d)** $\frac{4}{6}$ of $\frac{1}{2}$

4. What is the missing fraction in each sentence?

a) $\frac{2}{5}$ is $\frac{2}{3}$ of ▢ **b)** ▢ of $\frac{4}{9}$ is $\frac{1}{3}$

Lesson 2.3

5. Sketch a model for this calculation.

$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$

6. Calculate.

a) $\frac{2}{9} \times \frac{2}{7}$ **b)** $\frac{3}{7} \times \frac{2}{5}$ **c)** $\frac{5}{8} \times \frac{2}{3}$ **d)** $\frac{1}{5} \times \frac{5}{7}$

7. About $\frac{2}{3}$ of the students in Andee's school come by bus. About $\frac{1}{3}$ of these students are on the bus for more than an hour and a half each day. What fraction of the students in Andee's school are on the bus for more than an hour and a half each day?

Lesson 2.4

8. Which products are closer to $\frac{1}{2}$ than either 0 or 1?

a) $\frac{3}{4} \times \frac{5}{6}$ **b)** $\frac{1}{6} \times \frac{7}{8}$ **c)** $\frac{3}{9} \times \frac{8}{9}$ **d)** $\frac{3}{5} \times \frac{2}{3}$

Lesson 2.5

9. Draw two models to represent $1\frac{3}{4} \times 2\frac{1}{5}$.

10. Calculate. Express the answer as an improper fraction.

a) $2\frac{1}{3} \times \frac{3}{5}$ **b)** $3\frac{1}{4} \times 2\frac{2}{5}$

11. The supermarket has $2\frac{1}{2}$ times as many employees just before dinnertime as in the late morning. There are 18 employees in the late morning. How many employees are there just before dinnertime?

Lesson 2.6

12. Draw a diagram to show that $\frac{6}{8} \div 3 = \frac{2}{8}$.

13. Calculate.

a) $\frac{9}{10} \div 3$ **b)** $\frac{9}{10} \div 2$ **c)** $\frac{4}{5} \div 6$

14. Explain why $\frac{\square}{2} \div 3$ is a fraction that can be written with a denominator of 6.

Lesson 2.7

15. Which quotients are between 4 and 6?

a) $\frac{9}{10} \div \frac{1}{5}$ **b)** $\frac{9}{10} \div \frac{1}{8}$ **c)** $3\frac{1}{2} \div \frac{3}{4}$

16. What fractions might you use to estimate $\frac{7}{16} \div \frac{6}{20}$?

Lesson 2.8

17. Sketch a model to show $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$.

18. Explain how you know that $\frac{4}{6} \div \frac{3}{6}$ has the same quotient as $\frac{4}{5} \div \frac{3}{5}$.

19. Calculate.

a) $\frac{5}{6} \div \frac{1}{6}$ **b)** $\frac{5}{8} \div \frac{1}{4}$ **c)** $\frac{5}{6} \div \frac{1}{4}$ **d)** $\frac{3}{8} \div \frac{2}{9}$

20. What fraction calculation can you use to determine the number of quarters in $4.50?

Lesson 2.9

21. Pia used $\frac{2}{3}$ of her sugar to make $\frac{3}{4}$ of a batch of cookies. How much of her sugar would she have needed to make a whole batch?

Lesson 2.10

22. Which expression has the greater value? How do you know?

A. $\frac{4}{5} \times \frac{2}{3} - \frac{1}{5} \times \frac{5}{8}$ **B.** $\frac{4}{5} \times \left(\frac{2}{3} - \frac{1}{5}\right) \times \frac{5}{8}$

23. Where can you place brackets to make this equation true?

$3 \times \frac{2}{3} + \frac{1}{3} \div \frac{1}{4} = 12$

Lesson 2.11

24. Use fractions to explain why 4.5×0.5 equals 2.25.

Chapter Task

Task | *Checklist*

- ✔ Did you use appropriate strategies to compare file sizes?
- ✔ Did you use appropriate operations to compare file sizes?
- ✔ Did you use visuals, words, and symbols to explain your thinking?
- ✔ Did you explain your thinking clearly?

Computer Gizmos

Brian likes to write mini-applications for his computer. One application automatically displays a bar to show what fraction of a megabyte of memory a file is using at any point in time.

Megabyte Minder

File name	Memory (in MB)	
Science project		$\frac{1}{3}$
Journal		$\frac{7}{8}$
Short story		$\frac{2}{5}$
Book report		$\frac{5}{6}$

How can you describe how the sizes of Brian's files compare?

A. Which of Brian's files have room to be doubled before they reach the 1 MB mark? How do you know?

B. Compare the science project file to all the others by indicating what fraction of the larger file it is.

C. Compare the journal file to all the others by indicating what fraction of the smaller file it is.

D. How much memory is still available for the book report file compared to the science project file? Why is there more than one way to answer this question?

E. What other fraction comparisons related to these files can you make? Calculate the comparisons.